Hadron-Gluon Interactions and the Pomeron

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(June 22, 2001)

Abstract

A gauge invariant interaction term is added to the QCD Lagrangian involving hadron and gluon fields. An effective vertex is calculated for such interactions through exchanges of reggeized gluons. This gives rise to an effective coupling for hadron-gluon elastic scattering in the t-channel, which is used in an inclusive hadron-hadron interaction from which the Pomeron intercept $\alpha(0)_P$ is calculated.

11.55.J,12.40N

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I. INTRODUCTION

The QCD Pomeron and its Regge trajectory remains a formidable challenge in particle physics to this day. Perturbative QCD has led to the BFKL equation [1] for which solutions have been constructed to obtain Regge behavior in particular the intercept of the Pomeron trajectory $\alpha(0)_P$. With this approach the scattering is done at the quark level requiring that a minimum of two gluons be exchanged to first order, and that all exchanges be done between the same two quarks with a color singlet projected from the S matrix in each order. One may refer to this treatment of a singlet exchange as a hard Pomeron [7] because infra-red effects where $q < \Lambda_{QCD}$ are left out due to the non-perturbative nature of QCD at these energies. In hadron physics this means that one assumes the separation of the scattering quarks to be much smaller than hadronic lengths. Realistically, the dynamics of hadron scattering is such that IR effects play a crucial role in predicting the soft Pomeron [7] behavior for which an observed [6] trajectory gives $\alpha_P(0)$ at approximately 1.08. In a sense the Pomeron is an admission that perturbation has its limits in describing the compositeness of hadrons from constituent quarks, and it remains a challenge to formulate the interplay between the two. A more recent approach [3] has emerged in which it has been shown that QCD in the IR region is a diffeomorphism like interaction in which two gluon fields when contracted with their color indices make up a tensor $G\mu\nu = \delta^{ab}A^a_\mu A^b_\nu$ with properties similar to that of a metric tensor in gravity suggesting new gauge invariant interaction terms to be included in Lagrangians involving both gluons and hadron matter fields. This approach assumes no information on the scattering mechanism of the quarks inside the hadron but ensures that the hadron remains color neutral.

In the following analysis ideas from both approaches mentioned above will be incorporated to treat the problem using perturbation (in which case $q > \Lambda_{QCD}$) at the hadron level by introducing a gauge invariant term involving two gluon fields with derivatives, coupled to hadron fields. Although a BFKL equation will not be formulated at this stage we will consider reggeized gluon exchanges which will ultimately lead to Regge behavior of the S matrix. By introducing such interactions the model is an effective theory which circumvents the problem of specifying which quarks participate in the interactions, and thus there is no need to assume that scattering is done only between the same two quarks. The only stringent constraint is that the color neutrality of the hadron is preserved in the scattering process.

Our starting point is with the following Lagrangian density:

$$\mathcal{L} = \mathcal{L}_{QCD} + \mathcal{L}_{HG} \tag{1}$$

where the first term on the right denotes the full QCD Lagrangian, while the second term denotes an $SU(N)$ gauge invariant interaction term given by:

$$\mathcal{L}_{HG} = \frac{\lambda}{2}\bar{\phi}\delta^{ab}(\partial^\mu A^{\nu a} - \partial^\nu A^{\mu a})(\partial_\mu A^b_\nu - \partial_\nu A^b_\mu)\phi \tag{2}$$

In Eq. (2)[1] ϕ denotes a hadron matter field, λ is a coupling with dimension M^{-D}(where for

[1]Greek letter indices will be used to denote space-time, while small case Latin letter indices will

example a scalar meson $D = 2$ while for a spin $\frac{1}{2}$ baryon $D = 3$), $A^{\mu a}$ is a gluon field and δ^{ab} is the $SU(N)$ color metric. Since the coupling λ has negative mass dimension one should expect that at high energy it will decay, or in the language of renormalization group [11] $\mathcal{L}_{HG}$ consists of irrelevant operators which die away at relatively high energy. Since we are dealing with hadron interactions and the underlying theory is QCD then we should follow the energy scale defined by this theory which is Λ_{QCD}. Thus for perturbative purposes we may require that $M^D\lambda$ be very small or close to zero at this energy scale. In fact such a requirement is mandatory since amplitudes evaluated both as functions of λ and α_s should be finite as $q \to \Lambda_{QCD}$, or more simply put:

$$M^D\lambda \overset{q\to\Lambda_{QCD}}{=} \frac{C}{\alpha_s}$$

where C is some finite dimensionless constant.

II. HADRON-GLUON VERTEX

The Fynman rule for the hadron-gluon interaction follows from Eq. (2):

$$V^{\mu\nu ab} = i\lambda\delta^{ab}\triangle^{\mu\nu} \tag{3}$$

where

$$\triangle^{\mu\nu} = g^{\mu\nu}k\cdot k' - k^\nu k'^\mu \tag{4}$$

$V^{\mu\nu ab}$is the vertex shown in Figure 1. For simplicity the hadrons in (2) are chosen to be scalars, though this could be extended to any hadron of any spin with out changing the tensorial structure of the vertex. Also it is easy to see that this vertex is gauge invariant obeying a ward identity $k_\mu V^{\mu\nu ab} = 0$. This is crucial for ensuring the color neutrality of the hadron.

Our next step is to evaluate a correction to this vertex in perturbation theory where in an s-channel process of two gluons annihilating to two hadrons, a reggeized [2,5,4] gluon is exchanged in the t- channel between the two incoming gluons(figure 2). Eventually our interest will be in the t-channel process of elastic scattering of a hadron by a gluon, but as it turns out it is easier to first evaluate the s- channel process by using the Cutkosky rules [12], and then use crossing symmetry to get the t-channel amplitude[2].

The vertical line in the t- channel gluon shown in figure 2 denotes a reggeized gluon. The authors in references [2,5,4] have shown that in a QCD octet exchange in the Regge limit in which $\frac{t}{s} \ll 1$ the gluon sits on a Regge trajectory with an intercept $\alpha(0) = 1$. This amounts to replacing the gluon propagator in the t-channel with:

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be used to denote color of the gauge group. Also indices pertaining to symmetries such as flavor have been suppressed.

[2]One could have evaluated the t-channel loop correction from the start by introducing Fynman parameters, though this turns out to be cumbersome due to the derivative coupling of the vertex.

$$\frac{-ig^{\mu\nu}}{q^2} \Rightarrow \frac{-ig^{\mu\nu}}{\mathbf{q^2}}\left(\frac{s}{\mathbf{k^2}}\right)^{\epsilon(\mathbf{q})} \tag{5}$$

where

$$\epsilon\left(\mathbf{q}\right) = -\frac{N}{4\pi^2}\alpha_s \int \mathrm{d}^2\mathbf{k}\frac{\mathbf{q^2}}{\mathbf{k^2}(\mathbf{k}-\mathbf{q})^\mathbf{2}} \tag{6}$$

and $\mathbf{k}$ is a typical momentum scale at which gluons reggeize with the following conditions:

$$\Lambda_{QCD} < |\ \mathbf{k}\ | < \sqrt{s}$$
$$\frac{\mathbf{k^2}}{s} \ll 1.$$

For reggeization to occur, and for Eq. (5) to be true α_s must be fixed and less than unity. Since the Regge trajectory of the gluon is given by

$$\alpha(t) = \alpha(0) + \alpha' t, \tag{7}$$

it follows that the gluon sits on an infinite number of linear trajectories with the same intercept. This is because α' is in itself function of α_s which is a function of energy. It will be shown that summing on all these trajectories constitutes a part of a Pomeron exchange.

Proceeding to evaluate the amplitude using the Cutkosky cutting rules [12], the imaginary part of an amplitude is given by:

$$Im(A_{ab}) = \frac{1}{2}(2\pi)^4\delta\left(p_a - p_b\right)\sum_c A_{ac}A^\dagger_{cb} \tag{8}$$

where the sum is over intermediate states which are on mass shell and are denoted by cuts in the Fynman diagrams where those states appear. To obtain the real part of the amplitude one can utilize the analytical properties of the S matrix [7] in which it can be shown that if in an s-channel cut the imaginary part of the amplitude to n^{th}order is given by $A(\ln s)^n$, then by using the identity $ln(-s) = ln(s) - i\pi$ the real part of the amplitude is given by $-\frac{A}{\pi(n+1)}(\ln s)^{n+1}$ in leading logs.

For the s -channel process at hand the zeroth order amplitude (figure1) is purely real. The imaginary part of the amplitude of the first order correction is given according the cutting rules (figure3):

$$\begin{aligned} Im(A_{HG})_1 = \frac{1}{2}\sum_G \int \frac{\mathrm{d}^4\mathrm{u}}{(2\pi)^3}\frac{\mathrm{d}^4\mathrm{u}'}{(2\pi)^3}\delta(u^2)\delta(u'^2)(2\pi)^4 \\ \times\, \delta^4(k + k' - u + u')(A_{GG})_0(A_{GH})^\dagger_0 \end{aligned} \tag{9}$$

where $(A_{GG})_0$ is the amplitude left of the cut, and $(A_{GH})_0$ is the amplitude right of the cut in figure 3 respectively. The sum is over intermediate gluon polarizations. In the limit where the t-channel gluon with momentum q is mostly transverse and small compared to the center of mass energy, it is convenient to introduce **Sudakov parameters** and write q^μ as :

$$q^\mu = \xi_1 k_1^\mu + \xi_2 k_2^\mu + q_\perp \tag{10}$$

with ξ_1, ξ_2 being much less than unity. Thus to a good approximation it follows that:

$$q^2 = -\mathbf{q^2}.$$

With these simplifications the phase space of the integral in Eq. (9) becomes:

$$\begin{aligned}\int \frac{\mathrm{d}^4\mathrm{u}}{(2\pi)^3}\frac{\mathrm{d}^4\mathrm{u}'}{(2\pi)^3}\delta(u^2)\delta(u'^2)(2\pi)^4\delta^4(k+k'-u+u') \\ = \frac{s}{8(\pi)^2}\int \mathrm{d}\xi_1\mathrm{d}\xi_2\mathrm{d}^2\mathbf{q}\delta(-\mathbf{s}\xi_\mathbf{2}-\mathbf{q^2})\delta(\mathbf{s}\xi_\mathbf{1}-\mathbf{q^2})\end{aligned} \tag{11}$$

where one delta function has been absorbed, and second order terms in $\xi_1\xi_2$ have been dropped. The two remaining delta functions in the integrand above give the condition that $\xi_1 = -\xi_2 = \frac{\mathbf{q^2}}{s}$. Keeping in mind that q is small we proceed to approximate the tree level amplitudes in figure 3.

$$\begin{aligned}(A_{GG})_0 = 8\pi s\alpha_s g^{\mu\lambda}g^{\nu\sigma}\epsilon_\mu^{\alpha c}(u)\epsilon_\nu^{\beta d}(u')\epsilon_\lambda^{\gamma b}(k)\epsilon_\sigma^{\delta e}(k') \\ \times f^{abc}f^{ade}\left(\frac{1}{\mathbf{q^2}}\right)\left(\frac{s}{\mathbf{k^2}}\right)^{\epsilon(\mathbf{k})}\end{aligned} \tag{12}$$

$$(A_{HG})_0 = \lambda\delta^{b'e'}\triangle^{\nu'\mu'}\epsilon_{\nu'}^{\alpha e'}(k)\epsilon_{\mu'}^{\beta b'}(k') \tag{13}$$

with f^{abc} being the structure constants of the color group(the first four letters of the Greek alphabet denote gluon polarizations). Putting this together with Eq. (11) into Eq. (9) and summing over gluon polarizations we get the following expression for the imaginary part of the first order correction:

$$\begin{aligned}Im(A_{HG})_1 = -N\delta^{be}\epsilon_\lambda^{\gamma b}(k)\epsilon_\sigma^{\delta e}(k')\triangle^{\lambda\sigma}(k,k') \\ \times\frac{\lambda\alpha_s}{4\pi}\int \mathrm{d}^2\mathbf{q}\frac{1}{\mathbf{q^2}}\left(\frac{\mathrm{s}}{\mathbf{k^2}}\right)^{\epsilon(\mathbf{k})}\end{aligned} \tag{14}$$

We first treat the exponent in Eq. (14) given by Eq. (6). The integral in the latter is infra-red divergent, but can be regularized and redefined by introducing a scale to give: [3]

$$\epsilon(\mathbf{q}) = -\frac{N}{2\pi}\alpha_s\ln\left(\frac{\mathbf{q}}{\Lambda_{QCD}}\right) \tag{15}$$

Since $\epsilon(\mathbf{q})$ is a quantity evaluated purely using perturbation theory it is appropriate that the scale chosen be that of the natural scale set by the theory (QCD) which is Λ_{QCD}. If the expression for the running of α_s[4] is included as well, $\epsilon(\mathbf{q})$ becomes:

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[3]What has been done is to redefine $\epsilon(\mathbf{q}) \rightarrow \epsilon(\mathbf{q}) - \epsilon(\mathbf{\Lambda_{QCD}})$ as to eliminate dependence on regularization parameters which have no physical significance.

[4]The running of α_s can be included on the condition that the integral over $\mathbf{q}$ in Eq. (14) be limited to the perturbative region of QCD; namely from Λ_{QCD} and up. One cannot include the running in Eq. (6) since there $\mathbf{q}$ can take on any values including those not included in perturbative QCD, and therefore would make α_s diverge which would spoil the reggeization of the gluon.

$$\epsilon\left(\mathbf{q}\right) = -\frac{N}{b_o}. \tag{16}$$

b_o is the familiar constant given by

$$b_o = 11 - \frac{2}{3} n_f$$

and n_f is the number of flavors. The $\mathbf{q}$ dependence of ϵ has been eliminated, and Eq. (14) reads:

$$\begin{aligned} Im(A_{HG})_1 &= M(k_1,k_2)\left(\frac{s}{\mathbf{k^2}}\right)^{\epsilon}\int_{\Lambda_{QCD}}^{k} \mathrm{d}^2\mathbf{q}\frac{\lambda(\mathbf{q})\alpha_{\mathrm{s}}(\mathbf{q})}{\mathbf{q^2}} \\ &= M(k_1,k_2)\lambda_{eff}(s,\mathbf{k}). \end{aligned} \tag{17}$$

$M(k_1,k_2)$ is the tensorial product of the vertex with gluon polarization vectors times a color factor , and λ_{eff} is defined to be:

$$\lambda_{eff}\left(s,\mathbf{k}\right) = \left(\frac{s}{\mathbf{k^2}}\right)^{\epsilon}\lambda(\mathbf{k}). \tag{18}$$

In doing so we have redefined the vertex given in Eq. (2) to an effective vertex in which pure[5] QCD exchanges are integrated out. The real part of this amplitude is given by:

$$\begin{aligned} Re(A_{HG})_1 &= M(k_1,k_2) \\ &\times\left(-\lambda\left(\mathbf{k}\right) + \frac{\pi}{\epsilon}F\left(\mathbf{k}\right)\left(\left(\frac{s}{\mathbf{k^2}}\right)^{\epsilon(\mathbf{k})} - 1\right)\right) \end{aligned} \tag{19}$$

where $F(\mathbf{k})$ is the integral given by Eq. (17), and the zeroth order amplitude has been included since it is also first order in λ and contains no QCD exchanges. Similarly we want the real part of the amplitude to be proportional to $\lambda_{eff}(s,\mathbf{k})$ (up to a constant). Thus we ge the condition:

$$-\lambda\left(\mathbf{k}\right) + \frac{\pi}{\epsilon}\lambda_{eff}\left(s,\mathbf{k}\right) - \frac{\pi}{\epsilon}F\left(\mathbf{k}\right) = C\lambda_{eff}(s,\mathbf{k}). \tag{20}$$

Since both $\lambda(\mathbf{k})$ and $F(\mathbf{k})$ are independent of s, $\lambda(\mathbf{k})$ obeys the following integral equation:

$$\lambda\left(\mathbf{k}\right) = \frac{2\pi^2}{N}\int_{\Lambda_{QCD}}^{k} \mathrm{d}^2\mathbf{q}\frac{\lambda(\mathbf{q})}{\mathbf{q^2}\ln\left(\frac{\mathbf{q}}{\mathbf{\Lambda_{QCD}}}\right)} \tag{21}$$

with a solution of the form

$$\lambda\left(\mathbf{k}\right) = \frac{C}{M^D}\ln\left(\frac{\mathbf{k}}{\Lambda_{QCD}}\right). \tag{22}$$

Having found the effective vertex coupling we see that at least in orders of α_s, λ_{eff} is a decreasing function as s increases due to ϵ being negative conforming to the requirement that the amplitudes near Λ_{QCD} should approach a finite value.

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[5]By pure we mean interactions described only by the first term on the right side of Eq. (1.)

III. INCLUSIVE INTERACTIONS AND THE POMERON

As was mentioned in the beginning of section II our interest is in the elastic scattering of gluons and hadrons since these are assumed to constitute a major portion of the Pomeron. These t-channel amplitudes can now be easily obtained from the s-channel amplitudes calculated in section II by using crossing symmetry. In particular the expression for $\lambda_{eff}(t, \mathbf{k})$ becomes:

$$\lambda_{eff}\left(t, \mathbf{k}\right) = \left(\frac{t}{\mathbf{k}}\right)^{\epsilon} \lambda\left(\mathbf{k}\right) = \left(\frac{-2k_1 \cdot k_2}{\mathbf{k}}\right)^{\epsilon} \lambda(\mathbf{k}) \tag{23}$$

We consider now an inclusive hadronic interaction by which two hadrons with a center of mass energy $\sqrt{s}$ scatter through an exchange of a gluon in the t-channel plus outgoing jets (figure4).

Working in the center of mass frame (assuming again that our hadrons are scalar mesons of the same flavor) it is again convenient to introduce the **Sudakov parameters** and write the following vectors as:

$$k_1^\mu = \xi_1 p_1^\mu + \xi_2 p_2^\mu + k_{1\perp}^\mu \tag{24a}$$
$$k_2^\mu = \xi_1' p_1^\mu + \xi_2' p_2^\mu + k_{2\perp}^\mu \tag{24b}$$
$$k_3^\mu = \xi_1'' p_1^\mu + \xi_2'' p_2^\mu + k_{3\perp}^\mu \tag{24c}$$

where the vectors p_1, p_2 denote the momenta of the two incoming hadrons, k_1 is the momentum of the t-channel gluon, and k_2, k_3 denote the momenta of the emitted gluons which give rise to the jets [6] with amplitudes given by $L^{(2)}_{a\mu}, L^{(3)}_{b\mu}$. The following kinematic constraints are to hold:

$$p_{1\mu} p_1^\mu = p_{2\mu} p_2^\mu = 0 \tag{25a}$$
$$|\ \xi_1'\ | \gg |\ \xi_2'\ ||\ \xi_1\ ||\ \xi_2\ | \tag{25b}$$
$$|\ \xi_2''\ | \gg |\ \xi_1''\ ||\ \xi_1\ ||\ \xi_2\ | \tag{25c}$$
$$\xi_1 = -\xi_2 \tag{25d}$$
$$\xi_1' \approx -\xi_2''. \tag{25e}$$

The first condition just says that the incoming energy of the hadrons is much greater than their masses so that they can be neglected. The second and third conditions follow from the fact that k_2, k_3 are mostly along the beam line with very little transverse momenta, as so are the jets. The fourth condition says that k_1 is purely spatial, and mostly transverse, and the fifth condition is based on the assumption that the total incoming momentum of the hadrons is approximately equal to their outgoing, or in other words $k_2 \approx -k_3$. It is also assumed that all transverse momenta in (24) are very small compared to the incoming energy.

[6] It is possible with the interaction given in (2) two exchange two gluons in the t- channel and do away with the jets though this would seem to be less of a realistic scenario.

With these assumptions the tree level amplitude[7] (4) becomes:

$$A_{(HH)} = \delta^{ab}\lambda_{eff}(k_1,k_2)L^{(2)}_{a\mu}\triangle^{\mu\lambda}\left(\frac{g_{\lambda\sigma}}{k_1^2}\right)^{\epsilon} \times \lambda_{eff}(k_1,k_3)L^{(3)}_{b\nu}\triangle^{\sigma\nu} \tag{26}$$

Using statements (24) and (25) we have:

$$\lambda_{eff}(k_1,k_2) \approx \lambda_{eff}(k_1,k_3) = \lambda\left(\mathbf{k}\right)\left(\frac{\xi_1\xi_1' s}{\mathbf{k^2}}\right)^{\epsilon}$$

where both ξ_1 and ξ_1' are positive numbers. The tensorial product in Eq. (26) may be written as:

$$L^{(2)}_{a\mu}\triangle^{\mu\lambda}g_{\lambda\sigma}L^{(3)}_{b\nu}\triangle^{\sigma\nu} = L^{(2)}_{a\mu}L^{(3)}_{b\nu}\Big(g^{\mu\nu}(s\xi_1\xi_1')^2 - (s\xi_1\xi_1')k_1^{\nu}k_3^{\mu} - (s\xi_1\xi_1')k_1^{\mu}k_2^{\nu} + (k_2\cdot k_3)k_1^{\mu}k_1^{\nu}\Big).$$

Since the jet's final products are hadrons which carry no color, and due to the fact the in the present kinematic regime $k_2 \approx -k_3$, the second and third term on the right hand side in the equation above are proportional to $k_2^{\mu}L^{(2)}_{a\mu}, k_3^{\mu}L^{(3)}_{b\nu}$ which give zero as dictated by gauge invariance (up to first order in the **Sudakov parameters**). The fourth term is negligible since the vector k_1 is mostly transverse while the amplitudes $L^{(2)}, L^{(3)}$ are mostly longitudinal [8]; their contraction should approach zero. With this Eq. (26) reads:

$$\begin{aligned} A_{(HH)} &= \lambda^2\left(\mathbf{k}\right)\delta^{ab}g^{\mu\nu}L^{(2)}_{a\mu}L^{(3)}_{b\nu}\frac{(\xi_1\xi_1')^{2+2\epsilon}}{(\mathbf{k})^{4\epsilon}k_1^2}s^{2+2\epsilon} \\ &= f\left(k_1,k_2\right)s^{2+2\epsilon}. \end{aligned} \tag{27}$$

It is well known from Regge theory [10] that in the limit $s\to\infty$ the scattering amplitude takes the form

$$\mathcal{A} = f(t)s^{\alpha(t)} \tag{28}$$

where

$$\alpha(t) = \alpha(0) + \alpha' t \tag{29}$$

is the linear Regge trajectory of the particles exchanged. Using the Optical theorem it can be shown that the total cross section is given by:

[7]Although this is a tree level in our model(Eq. (2)) the amplitudes $L^{(2)}_a, L^{(3)}_b$ will contain higher order terms in QCD and will be imaginary. These amplitudes will be described by 'fan' diagrams in QCD. See [4]

[8]Since these jets are mostly along the beam line it is expected that they don't vary much with s, but with $k_\perp$

$$\sigma_{total} = \frac{1}{s} Im\mathcal{A}(s, t = 0) \sim s^{\alpha(0)-1} \tag{30}$$

Pomeranchuk's theorem [8,9] predicts that for $\alpha(0) > 1$ in which case the cross section rises with s, the particles exchanged carry quantum numbers of the vacuum, or what is known as the Pomeron where the net change in isospin is zero. While for the case $\alpha(0) < 1$ in which case the cross section drops with s the theorem predicts that the particles exchanged are mesons and there could be a net change of isospin through a scattering process. Since we have dealt all along with gluons we should be concerned with the former prediction. Comparing Eq. (27) with Eq. (28) it follows that[9]

$$\alpha(t) = 2 + 2\epsilon = 2 - \frac{2N}{11 - \frac{2}{3}n_f} = \alpha_P(0). \tag{31}$$

Where the subscript P denotes the Pomeron intercept. Putting $N = 3$, and $n_f = 6$ we get $\alpha_P(0) = 1.14$,and Eq. (30) gives

$$\sigma_{total} = f(k_1 = 0, k_2 = 0)s^{1+2\epsilon} = f(t = 0)s^{0.143} \tag{32}$$

according to the prediction of the Pomeranchuk theorem.

IV. CONCLUSIONS

First a few comments on the qualitative nature for our calculation of $\alpha_P(0)$. Eq. (14) can be looked at as summation on all possible reggeized gluons with momentum q with all possible Regge trajectories as one includes the running α_s as part of the integral. In particular Eq. (5) describes the propagator of a reggeized gluon which in our scheme is assumed to be soft and should be confined within the hadron. It is also true for the exponent $\epsilon(\mathbf{q})$ which is an integral over two soft gluons in an octet exchange [7] that are as well confined. In light of this it is appropriate that some mass parameter should have been included with these propagators as to describe their confinement within the hadron . In addition to this the running of the coupling α_s should have been included as well inside the integral of Eq. (6) to give a full description of the off shell gluons. Of course by not doing the above it was possible to get an expression for ϵ independent of $\mathbf{q}$, and thus it was possible to perform the integral Eq. (14). Had the mass parameter, and α_s been incorporated in the integrals mentioned above not only would the integration be a challenge, but there would be a mass parameter to struggle with for which at this stage QCD does not offer a way to predict its value. The measured value of $\alpha_P(0)$ is about 1.08 [6] and it seems that IR effects where

[9]The fact that $\alpha(t)$ is independent of t is just an indication that we have not been able to trace the entire trajectory to first order in our scheme. We expect (work in preparation) that higher order amplitudes in λ will lead to a BFKL equation [7] from which a trajectory will appear as a solution to the equation. In any case the behavior of the cross section at high energy depends only on $\alpha(0)$.

perturbation cannot be applied play a central role in understanding the Pomeron. In any case the final result for $\alpha_P(0)$ in our model would still take the form:

$$\alpha = 2 - \gamma \frac{2N}{11 - \frac{2}{3}n_f}$$

where γ is a numerical factor incorporating non-perturbative effects.

The dependence of $\alpha_P(0)$ Eq. (31) on the number of flavors has an interesting implication. As the number of flavors increases α drops. In fact increasing the number of flavors to seven would cause α to drop below 1. Having $\alpha < 1$ would clash with Pomeranchuk's theorem which guarantees that the cross section is an increasing function of s. In other words if we are to uphold the theorem even at very high energies, six flavors seems to be a crossroads in our model, since with the addition of one more flavor some new physics should appear to compensate for the drop in α. Whatever this new physics is (Supersymmetry?) it should involve particles that reggeize, and that play a major role in hadron interactions.

The reader may have taken notice that the interaction (2) is nothing more than the free kinetic term of the gluons in the pure QCD Lagrangian coupled to a hadron field. This conforms with our intuition since for a gluon to interact outside the hadron it should have momentum $q > m_\pi$ which is almost the same as saying $q > \Lambda_{QCD}$ in which case the gluon is free. It therefore follows that confined gluons similar to the ones considered in the loop correction giving rise to λ_{eff}, contribute more to the structure of the hadron and again contribute to the understanding the Pomeron intercept.

Finally the tensor given in (2) of gluons and their derivatives before contraction on its Lorenz indices has the form $R_{\mu\nu\rho\sigma}$. This tensor is antisymmetric on the exchange of μ and ν, or ρ and σ, but is symmetric with the exchange of $\mu\nu$ with $\rho\sigma$. These algebraic properties of a tensor describe a particle of spin 2 [13]. Eq. (31) would have given $\alpha = 2$ had we not incorporated loop corrections in our amplitude. This conforms to the model given in [3] which predicts a spin 2 behavior in QCD with no matter fields.

V. ACKNOWLEDGEMENT

I would like to thank professor Yuval Ne'eman for his helpful comments.

REFERENCES

[1] E. A. Kuraev, L. N. Lipatov, V. S. Fadin, *Sov.Phys.JETP* **45**, 199 (1977).

[2] E. A. Kuraev, L. N. Lipatov, V. S. Fadin, *Sov.Phys.JETP* **44**, 443 (1976).

[3] Y. Ne'eman, Dj. Sijacki, *International Journal of Modern Phyaics*, **A10**, 4399-4412 (1995).

[4] L. V. Gribov, E. M. Levin, M. G. Ryskin, *Physics Reports.* **100**, 3915 (1998).

[5] J. Bartels, *Nucl.Phys.* **B151**, 293 (1975).

[6] A. Donnachie, P. V. Landshoff, *Physics Letters* **B296**, 227-232 (1992).

[7] J. R. Forshaw and D. A.Ross, *Quantum Chromodynamics and the Pomeron*, (Cambridge University Press, New York, 1997)

[8] I. Ia. Pomeranchuk, *J. Exptl. Theoret. Phys.* **30**, 423 (1956).

[9] L. B. Okun', I. Ia. Pomeranchuk, *J. Exptl. Theoret. Phys.* **30**, 424 (1956).

[10] P. D. B. Collins, *Introduction to Regge Theory and High Energy Physics*, (Cambridge University Press, New York, 1977)

[11] M. E. Peskin, D. V. Schroeder, *Introduction to Quatum Field Theory*, (Addison-Wesley Publishing Company, 1995), pp. 399-403.

[12] R. E. Cutkosky, *J. Math. Phys.* **1**, 429 (1960).

[13] S. Weinberg *The Quantum Theory of Fields* (Cambridge University Press, New York, 1995), pp. 253.

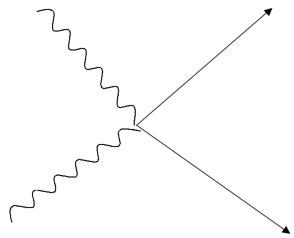

FIG. 1. Tree level amplitude for hadron-gluon scattering.

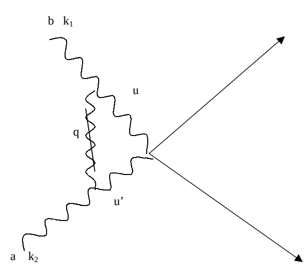

FIG. 2. First order amplitude with a reggeized gluon.

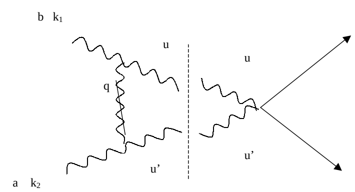

FIG. 3. Imaginary part of first order amplitude with a reggeized gluon.

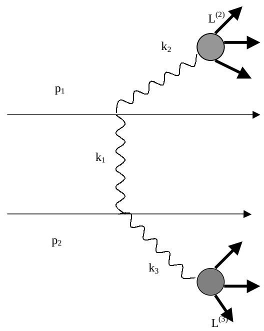

FIG. 4. Hadron to hadron scattering plus jets.